MECHANICS.

354. Proposed by G. PAASWELL, New York City.

The acceleration of an electric train is constant and equal to a feet per sec. per sec. Its braking or deceleration is variable and equal to the square root of the velocity. If the distance between stations is 5,000 feet, show that the acceleration must cease and braking ensue when the train is about 960 feet from the stopping point; also that the maximum velocity attained for a minimum time run is 88 m.p.h. and the time of run 54 seconds.

355. Proposed by HORACE OLSON, Chicago, Illinois.

A solid spheroid, axes a, a, b, is placed with its axis of revolution vertical. From its highest point, a particle is projected horizontally with a speed s. Where will it leave the spheroid, assuming that it slides on the surface without friction?

NUMBER THEORY.

272. Proposed by C. C. YEN, Tangshan, North China.

How many integers prime to n are there in each of the sets:

(a) $$1\cdot 2,\quad 2\cdot 3,\quad 3\cdot 4,\quad \cdots,\quad n(n+1);$$

(b) $$1\cdot 2\cdot 3,\quad 2\cdot 3\cdot 4,\quad 3\cdot 4\cdot 5,\quad \cdots,\quad n(n+1)(n+2);$$

(c) $$\frac{1\cdot 2}{2},\quad \frac{2\cdot 3}{2},\quad \frac{3\cdot 4}{2},\quad \cdots,\quad \frac{n(n+1)}{2};$$

(d) $$\frac{1\cdot 2\cdot 3}{6},\quad \frac{2\cdot 3\cdot 4}{6},\quad \frac{3\cdot 4\cdot 5}{6},\quad \cdots,\quad \frac{n(n+1)(n+2)}{6}?$$

273. Proposed by V. M. SPUNAR, Chicago, Illinois.

The ratio of the chances that all numbers ending in 1 or 9 and those ending 3 or 7 are composite is 3 : 2.

SOLUTIONS OF PROBLEMS.

ALGEBRA.

477. Proposed by J. L. RILEY, Junior College, Stephenville, Texas.

Evaluate the product $(1+r+r^2+r^3)(1+r^2+r^4+r^6)\cdots(1+r^{2^{n-1}}+r^{2^n}+r^{3.2^{n-1}})$.

Solution by Louis O'Shaughnessy, University of Pennsylvania.

Since

$$1+r+r^2+r^3=\frac{1-r^4}{1-r},\quad 1+r^2+r^4+r^6=\frac{1-r^8}{1-r^2},\quad 1+r^4+r^8+r^{12}=\frac{1-r^{16}}{1-r^4},\quad \cdots,$$

$$1+r^{2^{n-1}}+r^{2^n}+r^{3.2^{n-1}}=\frac{1-r^{2^{n+1}}}{1-r^{2^{n-1}}},$$

the required product is equal to the product of the following fractions:

$$\frac{1-r^4}{1-r}\cdot\frac{1-r^8}{1-r^2}\cdot\frac{1-r^{16}}{1-r^4}\cdot\frac{1-r^{32}}{1-r^8}\cdots\frac{1-r^{2^{n-1}}}{1-r^{2^{n-3}}}\cdot\frac{1-r^{2^n}}{1-r^{2^{n-2}}}\cdot\frac{1-r^{2^{n+1}}}{1-r^{2^{n-1}}}.$$

The result, manifestly, is

$$\frac{(1-r^{2^n})(1-r^{2^{n+1}})}{(1-r)(1-r^2)}.$$

Also solved by H. C. Feemster, Paul Capron, Elijah Swift, and Horace Olson.